WS #3-6
The Real Zeros of a Polynomial Function

1. You will be responsible to read the section completely and review the application of the following:

A. Division Algorithm for Polynomials : $\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$ or $f(x) = q(x)\cdot g(x) + r(x)$

B. Remainder Theorem : Let f be a polynomial function. If f(x) is divided by (x - c), then the remainder is f(c).

C. Factor Theorem : Let f be a polynomial function. Then (x - c) is a factor of f(x) if and only if f(c) = 0

1) If f(c) = 0, then x - c is a factor
2) If x - c is a factor of f(x), then f(c) = 0

D. Number of Real Zeros : A poly funct of degree n, (if n ≥ 1), that funct has at most n real zeros.

E. Rational Root Theorem : For poly functions of deg. 1 or higher. If $\frac{p}{q}$ in lowest terms, is a rational zero of f, the p must be a factor of a_0 & q must be a factor of a_n.

F. Every polynomial function (with real coefficients) can be uniquely factored into a product of linear factors and/or irreducible quadratic factors.

G. Bounds on Zeros :

2. Find the remainder if $f(x) = x^3 - 4x - 5$ is divided by:
A. x - 3 → remainder Theorem → $f(3) = (3)^3 - 4(3) - 5 = 10$; the remainder is 10 → f(c), where c = 3
B. x + 2 → " " → $f(-2) = (-2)^3 - 4(-2) - 5 = -5$; " " " -5 → f(c), where c = -2

3. Which of the following is a factor of $f(x) = 2x^3 - x^2 + 2x - 3$:
*use Factor Theorem
A. x - 1 → $f(1) = 2(1)^3 - (1)^2 + 2(1) - 3 = 0$, thus (x - 1) is a factor by factor theorem
B. x + 2 → $f(-2) = 2(-2)^3 - (-2)^2 + 2(-2) - 3 = -27 \neq 0$, thus x + 2 is not a factor.

4. For $f(x) = 2x^3 + 11x^2 - 7x - 6$ find:
A. The possible /potential rational roots → $\frac{p}{q} = \frac{\text{factors of } a_0}{\text{factors of } a_n} = \frac{\text{factors of } -6}{\text{factors of } 2} = \frac{\pm1, \pm2, \pm3, \pm6}{\pm1, \pm2}$
B. The actual rational zeros
↳ Graph in calc <on back>

So $\frac{p}{q} = \pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$

5. Find the real zeros of $f(x) = x^5 - x^4 - 4x^3 + 8x^2 - 32x + 48$. → graph in calc

<on back>

6. Solve the equation: $x^5 - x^4 - 4x^3 + 8x^2 - 32x + 48 = 0$ (Also write in factored form)
↳ solve is the same as finding the real zeros.

7. Find a bound to the zeros of each polynomial, then graph on the back of this sheet:
A. $f(x) = x^5 + 3x^3 - 9x^2 + 5$
B. $g(x) = 4x^5 - 2x^3 + 2x^2 + 1$

8. Find the real zeros of the polynomial function

$f(x) = x^5 - 1.8x^4 - 17.79x^3 + 31.672x^2 + 37.95x - 8.7121$

4 B.) After graphing in calc, we can see that -6 & 1 are potential zeros. Test $f(-6)$ first

$f(-6)=0$, so $x+6$ is a factor

use long or synthetic division to find remaining Factors:

$$\begin{array}{r|rrrr} -6 & 2 & 11 & -7 & -6 \\ & & -12 & 6 & 6 \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

$(x+6)(2x^2-x-1)=0$

try & factor

$2x^2-x-1 \rightarrow (2x+1)(x-1)=0 \rightarrow \boxed{x=-\frac{1}{2},\ x=1,\ x=-6}$

*If it doesn't factor easily, try quadratic formula to solve

5.) Find real zeros of $f(x)=x^5-x^4-4x^3+8x^2-32x+48$

Step 1: max # of zeros is $n \rightarrow 5$

Step 2: $\frac{p}{q} = \frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm16, \pm24, \pm48}{\pm1}$

Step 3: graph in calc

Step 4: test -3 first $f(-3)=0$, use synthetic division

$$\begin{array}{r|rrrrrr} -3 & 1 & -1 & -4 & 8 & -32 & 48 \\ & & -3 & 12 & -24 & 48 & -48 \\ \hline & 1 & -4 & 8 & -16 & 16 & 0 \end{array}$$

$(x+3)(x^4-4x^3+8x^2-16x+16)=0$

two is another potential zero looking at graph

$f(2)=0 \rightarrow$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline & 1 & -2 & 4 & -8 & 0 \end{array}$$

$(x+3)(x-2)(x^3-2x^2+4x-8)=0$

$x^3-2x^2+4x-8=0$

$x^2(x-2)+4(x-2)=0 \rightarrow (x^2+4)(x-2)=0$

2 non-real solutions

$x=2$

real zeros: -3 & 2 w/multiplicity 2

Factored Form: $(x+3)(x-2)^2(x^2+4)$